PhD – Accelerated Diffusion Processes for Uncertainty Quantification in Image Reconstruction based on a Low Rank Model

Problematic – Inverse problems, *machine learning*, high dimensions, image reconstruction, denoising diffusion probabilistic model.

Tools – DNN, DDPM, regularization, sampling, optimization.

Applications – Deconvolution, Fourier synthesis.

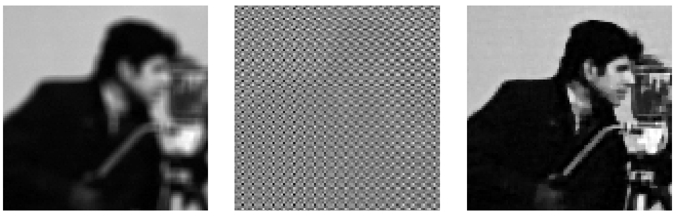
Contact and loc. – F. Orieux (francois.orieux@universite-paris-saclay.fr, 01 75 31 70 55), L2S laboratory. J. Idier, LS2N, jerome.idier@ls2n.fr.

Start of funding – October 1, 2025.

Context

Processing instrumental measurements often requires the use of the data model and the characterization of the **uncertainty** attached to the measurement. For instance, measurements are affected by noise or blurring, or live in a space other than that of the unknowns (Fourier coefficients *versus* an image in the case of MRI or interferometry).

While the direct model is stable and well-posed (data can be generated from the parameters), the inverse problem is often unstable and ill-posed [1], as illustrated in the figure (1b) where the result is unsatisfactory.



(a) Acquired image (b) Inverse filter inverse (c) Deconvolution

FIGURE 1 – An example of an ill-posed inverse problem : deconvolution. (1b) is the output of a naive reconstruction without regularization.

The subject is in the context of a new collaboration between *Laboratoire des Signaux et Systèmes* (L2S) and *Laboratoire des Sciences du Numérique de Nantes* (LS2N), both laboratory with recognized expertises in Bayesian inference for inverse problems.

Subject

Techniques for solving inverse problems have evolved considerably in recent years, with new machine learning techniques [2]. We can mention the unrolling of iterative algorithms [3], *plug-and-play* approaches or methods based on data-driven *prior*. Denoising Diffusion Probabilistic Model (DDPM) [4], [5] is notable in the sense that the process learn a probabilistic model like poserior law $q(x) = p(x \mid y)$, via Markov Chain modeling and Variationnal Bayesian estimation, to match an empirical distribution. This kind of model has been first proposed for image generation, but it can be seen as more generic to sample from any distribution, since the learning is complete.

In the context of inverse problems and Bayesian approach, the problem of reconstruction can be stated as the posterior mean estimator

$$\hat{x} = \int_x x \int_\theta p(x, \theta \mid y) \, \mathrm{d}\theta \, \mathrm{d}x \tag{1}$$

where θ are hyperparameters and x is the unknown of interest. A great advantage of the Bayesian framework is the natural **characterization of the uncertainty with higher-order moments**. However, the problem of posterior mean computation is challenging as it requires the evaluation of an integral in high-dimension with complex data adequacy $p(y \mid x, \theta)$ and prior model $p(x, \theta)$. In [6] and [7], we proposed efficient algorithms PO and RJPO to sample from posterior distributions by using specific structures often encountered in inverse problems and **perturbation injection like in diffusion model**. However, such algorithms are still too slow to process large volume data Markov Chain Monte Carlo (MCMC) diffusion based is a promising tool.

Description of the work

— First, the PhD student will review state-of-the-art data-driven and statistical learning methods for solving inverse problems, focusing on recent learning methods involving *Plug-and-Play* and DDPM.

— Next, the student will consider previous works on PO and RJPO and will make connections with diffusion processes used in DDPM based methods. Since PO and RJPO rely on linear system resolution, **low-rank** approximation of the model will be explored to accelerate the algorithm through preconditionning techniques based on specifically devised preconditionners. Ng and Pan's pioneering work [8] will serve as a starting point to deal with the deconvolution of sparse objects. The work will then be extended to **diffusion models based on more general prior models**.

— The aim is to identify the contributions and limitations of these approaches for **uncertainty quantifica-**

tion for inverse problems, and to propose possible solutions to the problems encountered. In particular, we could explore several types of preconditionners.

- The proposed methods will be implemented, and the results will be carefully compared with conventional and state-of-the-art approaches for which codes are available.
- The illustration will be addressed in the team application such as classical deconvolution, Fourier synthesis in radio astronomy (SKA project) or microscopy.
- The work will be carried out on a workstation equipped with a Nvidia 3080 or 4090 GPU with Linux, TensorFlow and Python or the Ruche cluster of Paris-Saclay University.

Profile

The candidate should have an engineer or a Master 2 degree in signal or image processing, data science or machine learning. Knowledge of applied mathematics or programming is required. Skills in estimation and statistics will be appreciated.

EU COFUND DeMythif.AI program

This PhD topic is participating to the Université Paris-Saclay EU COFUND DeMythif.AI program : `https://www.dataia.eu/appels-sprojets/ouvert-scofund-sdemythifai-sappel-ssujets-sde-sthese-s2024`. It is especially advertised to international students who have spent less than 12 months in France in the last 3 years. The candidates will be evaluated by a jury who will select 20 PhD to start in fall 2025. The successful candidates will be fully funded for 3 years (monthly gross salary 2 430 €, mobility indemnities 3 000€ and financial support to the host laboratories 10 000 €), have access to specific scientific and non-scientific training, and be fully part of the Université Paris-Saclay AI community

Références

[1] J. Idier, « Convex Half-Quadratic Criteria and Interacting Auxiliary Variables for Image Restoration, » *IEEE Trans. on Image Process.*, t. 10, n° 7, p. 1001-1009, juill. 2001.

[2] C. M. Bishop, *Pattern Recognition and Machine Learning* (Information Science and Statistics). New York : Springer, 2006, 738 p.

[3] D. Gilton, G. Ongie et R. Willett, « Deep Equilibrium Architectures for Inverse Problems in Imaging, » *IEEE Transactions on Computational Imaging*, t. 7, p. 1123-1133, 2021.

[4] J. Ho, A. Jain et P. Abbeel, « Denoising Diffusion Probabilistic Models, »

[5] C. A. Bouman et G. T. Buzzard, « Generative Plug and Play : Posterior Sampling for Inverse Problems, » in *2023 59th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, sept. 2023, p. 1-7.

[6] F. Orieux, O. Féron et J.-F. Giovannelli, « Sampling High-Dimensional Gaussian Distributions for General Linear Inverse Problems, » *IEEE Signal Processing Letters*, t. 19, n° 5, p. 251-254, 2012.

[7] C. Gilavert, S. Moussaoui et J. Idier, « Efficient Gaussian Sampling for Solving Large-Scale Inverse Problems Using MCMC, » *IEEE Transactions on Image Processing*, t. 63, n° 1, p. 11, 2015.

[8] M. K. Ng et J. Pan, « Approximate Inverse Circulant-plus-Diagonal Preconditioners for Toeplitz-plus-Diagonal Matrices, » *SIAM J. Sci. Comput.*, t. 32, n° 3, p. 1442-1464, jan. 2010.